ASSESSING ORBIT DETERMINATION REQUIREMENT WITH UNSCENTED TRANSFORMATION: CASE STUDY OF A LUNAR CUBESAT MISSION

Sun Hur-Diaz and Ravishankar Mathur

Emergent Space Technologies, Inc.

ABSTRACT

The orbit determination (OD) requirement for a low-thrust lunar CubeSat mission is examined. One driver for the OD accuracy is its contribution to the delta-V budget and hence the spacecraft's ability to achieve the target lunar orbit. Typically, this type of analysis is done using Monte Carlo simulations, but the large number of cases required to achieve a statistically significant result is often prohibitive. In this paper, we examine the use of unscented transformations to determine the impact of OD accuracy on the delta-V budget. This method is not unlike the linear covariance analysis; however its use of sigma points extends its usefulness beyond the linear region, especially for the highly nonlinear problem of the low-thrust transfer to the Moon. The efficacy of the unscented transformation method is demonstrated by comparing the results of this technique with the results from a small-scale Monte Carlo simulation and linear covariance analysis.

Index Terms— orbit determination, unscented transformation, delta-V budget

1. INTRODUCTION

The motivation for this study is the NASA CubeQuest Challenge [1], for which the winning 6U CubeSats in the competition are offered a launch on the Exploration Mission (EM) 1 mission as secondary payloads. All such CubeSats will be disposed into a high-energy trajectory that will fly by the Moon. Unless high-impulse chemical propulsion system is allowed on the CubeSats, most designs will involve some form of a low-thrust propulsion system to achieve lunar orbit. In order to determine the OD strategy for such a mission, the OD accuracy requirement needs to be understood.

This paper describes a method of evaluating the OD accuracy based on unscented transformation [2] of the position and velocity uncertainties to changes in the delta-V. The resulting sensitivity can be used to derive either the OD accuracy requirements or the delta-V requirements.

We first describe the representative lunar CubeSat mission followed by a description of how OD accuracy affects the delta-V. For illustrative purposes, we focus on the transfer trajectory of the mission and describe the nominal OD strategy utilized for this study. The equations associated with the unscented transformation, the Monte Carlo simulation, and the linear covariance analysis are presented followed by a description of the simulation set up. The simulation results and conclusions complete this paper.

2. LUNAR CUBESAT MISSION

The lunar CubeSat mission consists of the transfer trajectory to a lunar capture orbit, the spiral down to the target mission lunar orbit, the maintenance of the target orbit, and the disposal orbit. In this paper, we focus on the transfer orbit. We consider an extremely low thrust level of 1 mN for the propulsion system. Hence the transfer is designed to be one that is low-energy and takes advantage of the multi-body dynamical system consisting of the Earth, the Moon, and the Sun. The details of the trajectory design is provided separately [3]. Figure 1 depicts a nominal transfer trajectory consisting of the three primary burns summarized in Table 1. The first burn occurs before the high-energy lunar flyby, the second is a very long burn that occurs near the Sun-Earth L1 libration point, and the third is a maneuver that injects the CubeSat into a stable distant retrograde orbit (DRO) capture about the Moon.

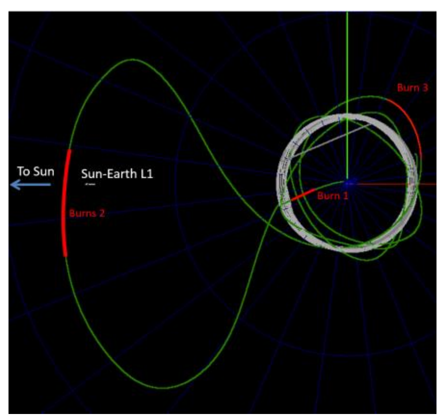

Figure 1. Nominal low-energy transfer trajectory

Table 1. Summary of the transfer trajectory maneuvers

Maneuver Name	Delta-V (m/s)	Duration (days)
Pre-Flyby	14.2	2.3
Sun-Earth L1	215.9	35.0
Lunar DRO	43.2	7.0

For the final mission design, additional trajectory correction maneuvers (TCMs) may also be incorporated.

3. DELTA-V BUDGET

The delta-Vs shown in Table 1 are only nominal. In actual operations, there are many sources of error that force us to deviate from the nominal including initial condition error in deploying the CubeSat from the launch vehicle upper stage, orbit determination error, spacecraft pointing error, propulsion error, unknown forces acting on the spacecraft, and deviations in spacecraft mass properties. Therefore, it is important to allocate a budget item for each of these as well as an additional overall margin for unknowns and contingencies. We focus on the effect of the OD error.

Figure 2 shows the different trajectories associated with this problem. The bold curve is the nominal trajectory used to compute the nominal maneuvers. The true trajectory, the thin curve, has a different initial condition due to the launch vehicle dispersion during separation. The estimated trajectory, the dashed curve, is our knowledge of the orbit obtained from filtering measurements associated with the true trajectory. The planned maneuver is based on the estimated trajectory, and the true maneuver is the planned maneuver applied to the true trajectory with execution error consisting of pointing error and propulsion error.

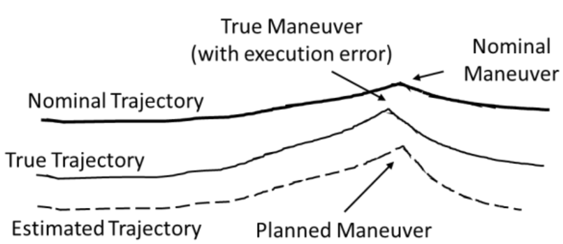

Figure 2. Various notions of trajectory

To understand how OD error affects the delta-V budget, we describe a closed-loop operational sequence. Sometime before a nominal burn, we obtain an estimate of the spacecraft position and velocity from which we ascertain our ability to reach our target if the nominal maneuvers were applied. If not, we re-plan the maneuvers from that point on, either to get back on the original nominal trajectory or a new trajectory.

One can think of the spacecraft state *x*, consisting of its position and velocity, as a random variable with a mean $\overline{x}$ and covariance P_{xx}. The delta-V required to achieve the lunar transfer can be thought of as another random variable, *u*, that is related to the state random variable, *x*, through a nonlinear function *f( )* as shown:

$$u = f(x) \tag{1}$$

with mean $\overline{u}$ and covariance P_{uu}. This covariance represents the extra delta-V due to OD error.

4. LINEAR COVARIANCE ANALYSIS

If we take just the first-order approximation of (1), the delta-V covariance due to OD error can be denoted as follows:

$$P_{uu}(i) = \frac{\partial(u_i)}{\partial(\hat{x}_i)} P_{xx}(i) \left(\frac{\partial(u_i)}{\partial(\hat{x}_i)} \right)^T \tag{2}$$

where $\hat{x}_i$ is the estimated state, with the corresponding state uncertainty covariance of $P_{xx}(i)$ (also obtained from OD) and is used in the planning of the *i*-th maneuver u_i, and $\frac{\partial(u_i)}{\partial(\hat{x}_i)}$ is the sensitivity of the planned maneuver to the estimated state. The linearization of the maneuver function can lead to significant biases or errors.

Figure 3 shows a "black box" showing the maneuver function, which takes as input the estimated state and outputs the planned maneuver.

Figure 3. Maneuver function "black box"

Typically, as it in this low-thrust lunar transfer problem, the "black box" is very complicated and involves complex nonlinear functions, including iterative functions. Hence, it is rare that we can derive an analytical expression of the maneuver sensitivity matrix. Alternatively, it can be approximated by numerical differencing of the maneuver function:

$$\frac{\partial(u_i)}{\partial(\hat{x}_i)}(:, j) = \frac{u(i, j) - u(i)_{NOM}}{\hat{x}_j(i) - \hat{x}_j(i)_{NOM}} \tag{3}$$

where $\hat{x}_j(i)$ is the *j*-th element of the perturbed estimated state and $u(i, j)$ corresponds to the output of the controller for that particular input state with the *j*-th element perturbed. The step sizes can be determined from iteration, and if the system is linear, the sensitivity matrix is not as sensitive to the step size. If the system is nonlinear, however, it can be quite sensitive. The use of unscented transformation ameliorates this problem since the Jacobian matrix is not required and can provide a more accurate statistics of the maneuver variation.

Although not considered in this paper, if we were to also consider the effect of the execution error, then Eqn. 2 would be pre- and post-multiplied by the variation of the actual maneuver v_i due to the execution error on the planned maneuver u_i:

$$\frac{\partial(v_i)}{\partial(u_i)} \tag{4}$$

A more general application of the navigation noise is given by Geller [4] where an augmented system consisting of both navigation and control is considered.

5. UNSCENTED TRANSFORMATION

A complete description of unscented transformations is given by Julier [2]. The unscented transformation is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. Here we summarize in the context of transforming the uncertainty of the state to the variation in the planned maneuver. In unscented transformation, the state vector and its error covariance are used to create a collection of 2L+1 representative vectors with the same statistical properties as the state and state error covariance, where L is the number of states. These equivalent state vectors are referred to as sigma points with an assigned weight for each vector. The individual vectors are propagated through the complete nonlinear function of the system to yield a cloud of the transformed points, which in this case are the delta-V maneuvers. Our system is represented by the maneuver planner black box shown in Fig. 3. The assigned weights are used to combine the transformed points to form its mean and covariance. Note that unlike Monte Carlo methods where a large number of samples are drawn at random, the unscented transformation method captures the high order information about the distribution using only a small number of points that are selected based on a specific, deterministic algorithm.

The 6-dimensional spacecraft state random variable with the nominal trajectory state as the mean and its covariance from the OD process,

$$P_{xx} = E\left[(x-\bar{x})(x-\bar{x})^T\right] \tag{5}$$

is approximated by a set of 13 sigma points:

$$\chi = \left[\bar{x} \quad \bar{x} \pm \left(\sqrt{(L+\lambda)P_j}\right)\right] \quad \text{for } j = \{1,...,L\} \tag{6}$$

where $\sqrt{P_j}$ is the j-th column of the Cholesky decomposition of the covariance matrix, and λ is a scaling parameter. The total number of sigma points is 2L+1 where L is the number of states.

For the propagation, each sigma point state is assigned a weight according to

$$W_0^m = \frac{\lambda}{L+\lambda} \tag{7}$$

$$W_0^c = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta) \tag{8}$$

$$W_k^m = W_k^c = \frac{1}{2(L+\lambda)} \tag{9}$$

where

$$\lambda = \alpha^2(L+\kappa) - L \tag{10}$$

with α controlling the spread of the sigma points, β incorporating distribution knowledge and κ as a secondary scaling parameter. The sigma points are propagated through the system, i.e., the black box. Process noise covariance can be included, if desired.

The resulting maneuver and its covariance are obtained by combining the set of the maneuver sigma points $u_{i,k}$ with their appropriate weights yielding

$$\bar{u}_i = \sum_{k=0}^{2L} W_k^m u_{i,k} \tag{11}$$

$$(P_{uu})_i = \sum_{k=0}^{2L} W_k^c (u_{i,k} - \bar{u}_i)(u_{i,k} - \bar{u}_i)^T \tag{12}$$

6. SIMULATION

To assess the effectiveness of the unscented transformation method, we compare it with the linear covariance method and the Monte Carlo method. We make this comparison for the Pre-Flyby maneuver and the Earth-Sun L1 maneuver to determine how the OD error that is obtained before the Pre-Fly maneuver affects both of these maneuvers. Because the trajectory design process fixes the Lunar DRO maneuver [3], it does not change with the Pre-Flyby OD error.

6.1. Simulation Setup

Two open-source software tools from NASA Goddard Space Flight Center are utilized to perform this analysis: the General Mission Analysis Tool (GMAT) [5] is used for the low-thrust maneuver planning in the “black box,” and the Orbit Determination Toolbox (ODTBX) [6] is used for performing the orbit determination. The OD problem is set up with ground stations as shown in Figure 4. Several OD strategies were considered including 2-way range measurements from the three primary stations indicated as VALP, KLNP, and DS46 as well as a backup strategy where only the US stations are utilized with 1-way range rate measurements. In this paper, we show only the latter strategy where 1-way range rate measurements are simulated with 1-σ noise of 15 mm/s and a range rate bias of 1 km/s.

Figure 4. Ground station locations

The tracking schedule for the first maneuver is such that we start tracking as soon as able after spacecraft disposal from the launch vehicle upper state and initial spacecraft checkout, at one minute interval, until about 10 hours before the start of the nominal maneuver. The 10 hours provides sufficient time to re-plan maneuvers if needed and for contingencies. The station visibility from the time of CubeSat disposal from the EM-2 upper stage to the start of the nominal first maneuver is shown in Figure 5.

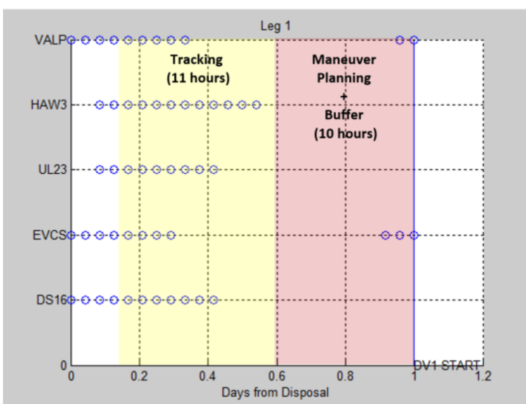

Figure 5. Station visibility at 1-hour increments

The "black box" that generates the maneuver plan from the OD estimate is implemented using GMAT as a variation of the nominal trajectory design process. Specifically, GMAT forward-propagates from the initial conditions, backward-propagates from the desired lunar arrival orbit, and uses the Sparse Nonlinear Optimizer (SNOPT) to compute optimal values for the first and second burn parameters. This process is modified to incorporate OD results by inserting a state perturbation at the designated plan time (10 hours prior to the first burn). SNOPT is once again used to optimize the burn parameters, using their nominal trajectory values as an initial guess. Because the OD-imposed perturbations are relatively small in magnitude, it can safely be assumed that the new optimal burn parameters will be close to their nominal values. Therefore, to reduce the time required to run many simulations, SNOPT is run in targeting-only mode and quickly returns a continuous trajectory with suboptimal (but still very close to optimal) values for the burn parameters. These parameters are used to convert the finite burns to their equivalent impulsive Delta-V vectors using the force-momentum relationship:

$$\Delta\bar{\boldsymbol{V}} = \left({}^{T\Delta t}/_{m}\right)\hat{\boldsymbol{u}}$$

Where Δt and $\hat{\boldsymbol{u}}$ are the thrust duration and unit-vector direction as computed by the simulation, T is the constant thrust magnitude (1mN), and m is the mass (14kg).

6.2. Monte Carlo (MC) Simulation

The sequential estimator, *estseq*, from ODTBX, is used to estimate position, velocity and the range rate bias between the onboard clock and the ground reference (typically GPS time). The results of a 10-case Monte Carlo simulation of the OD are shown in Fig. 6.

The state error covariance from the OD process at the start of the maneuver planning window is shown below:

$$P_{xx}(1) = \begin{bmatrix} 0.4 & .07 & 0.6 & 9\times10^{-6} & 3\times10^{-6} & 7\times10^{-6} \\ & 0.2 & -1 & -3\times10^{-6} & 7\times10^{-6} & -1\times10^{-5} \\ & & 20 & 1\times10^{-4} & -4\times10^{-5} & 2\times10^{-4} \\ & \cdots & & 6\times10^{-10} & -1\times10^{-10} & 9\times10^{-10} \\ & & & & & -3\times10^{-10} \\ & & & & 2\times10^{-10} & \\ & & & & & 2\times10^{-9} \end{bmatrix}$$

The uncertainties in the OD solution is the square root of the diagonal elements:

$$\delta x = [0.63 \quad 0.49 \quad 4.9\,]\ \text{km}$$
$$\delta v = [25 \quad 15 \quad 47]\ \text{mm/s}$$

Each OD solution is used as input in the "black box" to generate the corresponding Pre-Flyby and the Sun-Earth L1 maneuvers. The delta-V vectors are saved for each *j*-th MC run. The maneuver mean and covariance over *N* number of runs for a given maneuver *i* is computed as follows:

$$\bar{u}(i) = \frac{\sum_{j=1}^{N} u_j(i)}{N}$$
$$P_{uu}(i) = \frac{\sum_{j=1}^{N} (u(i)-\bar{u}(i))(u(i)-\bar{u}(i))^T}{N} \quad (13)$$

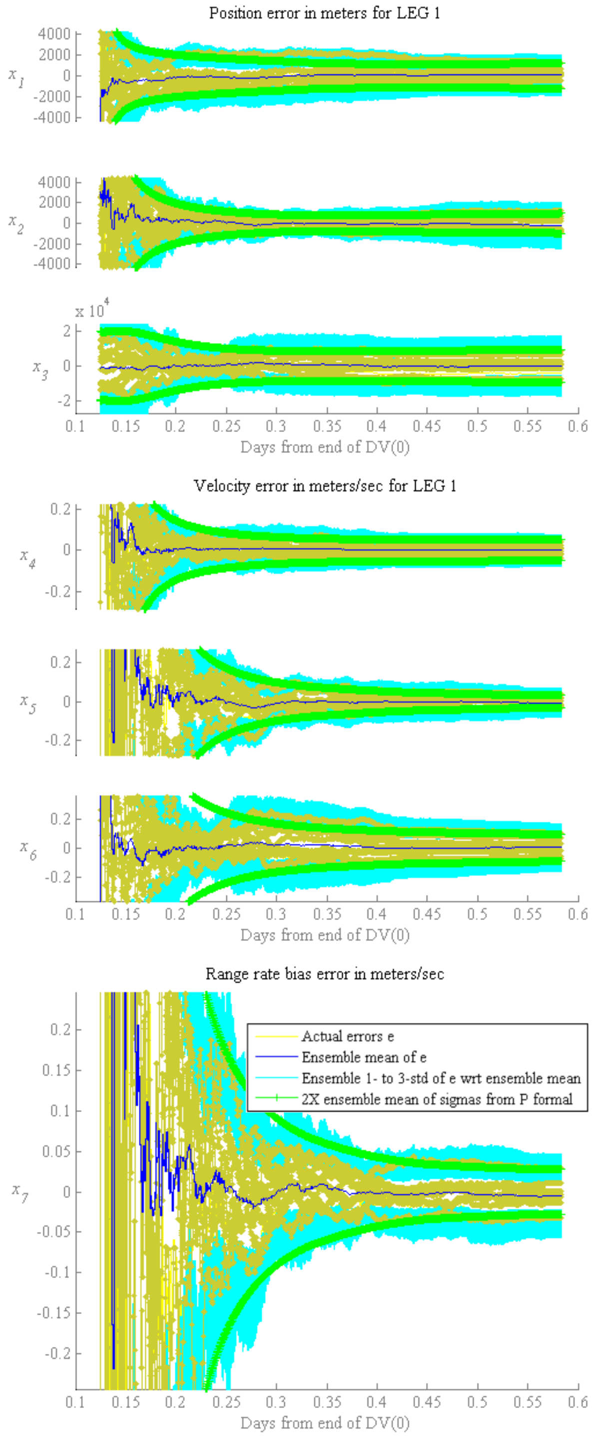

Figure 6. State estimation errors, their formal covariances, and the ensemble mean covariances from a 10-case Monte Carlo simulation of the Pre-Flyby OD. The dark solid line in each plot is the ensemble mean of all the estimate error.

6.3. Linear Covariance (LC) Method

The linear covariance method involves computing the maneuver sensitivity matrix. As stated previously, because there is no analytical expression for how GMAT solves for the planned maneuvers, the maneuver sensitivity matrix is numerically derived as shown in Eqn. 3. For the position states, the step size is chosen to be 1 km. For the velocity state, the step size is chosen to 50 mm/s. The nominal state at the maneuver planning time is perturbed by these steps to compute the maneuver sensitivity to the state at the maneuver planning time. The navigation covariance matrix is pre- and post-multiplied by this sensitivity matrix to form the maneuver covariance matrix.

6.4. Unscented Transformation (UT) Method

From the covariance matrix obtained from the OD, the 13 sigma points and their weights are generated. The UT parameters were chosen as follows:

$$\alpha = 1,\ \beta = 0,\ \text{and}\ \kappa = -3$$

The resulting sigma points in position and velocity are shown as squares in Fig. 7. These sigma points are processed by the “black box” to generate the 13 corresponding sigma points associated with each of the two maneuvers. These are combined as shown in Eqns. 11-12 to form the maneuver covariance matrix.

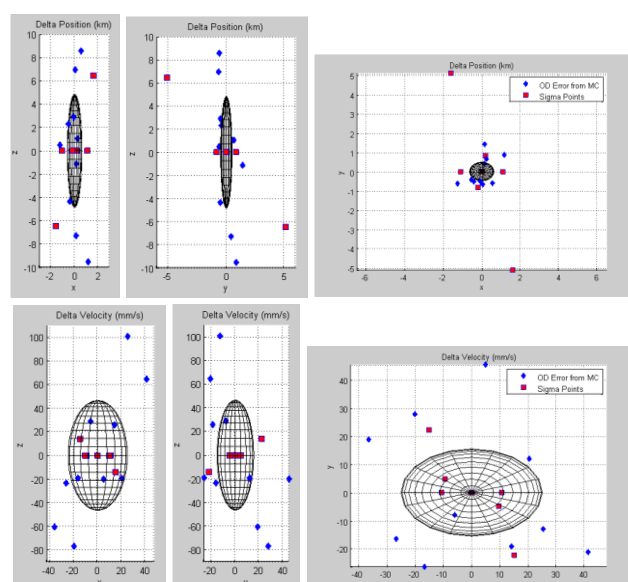

Figure 7. Different views of the position sigma points (top 3 plots) and the velocity sigma points (bottom 3 plots) relative to the formal OD covariance obtained from the OD process.

Also shown are the ten OD estimates from the Monte Carlo simulation relative to its mean.

6.5. Results

The results of the three methods are summarized in Tables 2 and 3.

Table 2. Pre-Flyby maneuver variations (m/s) based on the Pre-Flyby OD uncertainties

Method	δ(ΔVx)	δ(ΔVy)	δ(ΔVz)	|δΔV|
MC	0.74	0.04	0.18	0.76
LC	0.66	0.05	0.17	0.68
UT	0.73	0.08	0.17	0.75

Table 3. Sun-Earth L1 maneuver variations (m/s) based on the Pre-Flyby OD uncertainties

Method	δ(ΔVx)	δ(ΔVy)	δ(ΔVz)	|δΔV|
MC	0.36	0.94	0.68	1.22
LC	0.32	0.85	0.62	1.10
UT	0.36	0.93	0.68	1.21

The results indicate that the unscented transformation method matches the Monte Carlo better than the linear covariance method by about 10%.

It is interesting to note that the OD strategy for the Pre-Flyby OD used in the simulation increases the delta-V of each maneuver by only about 1 m/s (1-σ). If there is room in the delta-V budget, a lower OD accuracy may be acceptable with a possibility of relaxing the tracking schedule or the number of ground stations for the Pre-Flyby OD to reduce operational cost. Note that a similar analysis should be performed for OD done before other transfer maneuvers.

7. CONCLUSIONS

We examined three methods for determining the impact of OD accuracy on the delta-V budget. A CubeSat mission to the Moon, requiring very low-thrust, long duration maneuvers, was used as an example. The three methods were unscented transformation, linear covariance, and Monte Carlo. OD simulation results corresponding to the Pre-Flyby maneuver was used to do the comparison. It was found that the unscented transformation method matches the 10-case Monte Carlo result better than the linear covariance method by about 10%. For a better statistical representation, the number of Monte Carlo cases should be much higher. However, the unscented transformation method can be used, at a much less computational cost, to assess the maneuver variation due to OD error. This in turn can help determine the OD accuracy requirement to stay within the delta-V budget. One can vary the OD strategy such as number of stations, measurement type, and tracking schedule to determine if the resulting OD accuracy is commensurate with the delta-V budget.

8. REFERENCES

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