

ON-BOARD ESTIMATION OF COLLISION PROBABILITY FOR CLUSTER FLIGHT

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Being able to detect and respond to potential collisions is a significant concern for satellite cluster flight. The accuracy and speed at which collision probability is estimated is a key factor in determining the minimum allowable closest approach distance between any two modules, and thus the total size of the cluster. Perhaps the simplest metric of collision probability is propagating the navigation state forward in time and calculating the closest approach distance. While this method has the advantages of being simple and easy to implement onboard, it does not take into account the uncertainty in the navigation state. A method is presented which takes into account both the state and state uncertainty to estimate collision probability. Additionally this method generates a range of values in which the true probability of collision will fall. The accuracy of this method is verified through Monte Carlo simulation.

INTRODUCTION

Spacecraft collision probability is a growing area of interest. Several key factors drive this, among which is a growing desire to perform proximity operations as well as missions involving cluster flight. For cluster flight the risk of collision generally decreases as the intermodule distance increases or the cluster size increases. In order to decrease the probability of a collision, P_c , occurring within a cluster one could simply increase the separation between modules. However a larger cluster may lead to adverse side effects such as increased fuel usage and communication power requirements. As a result, there is a need to have a quick, effective method for computing collision probability in order to allow the modules to safely operate without unduly increasing the cluster size.

Several definitions are important to the concepts in this paper and to collision probability in general. The first is conjunction. Conjunction occurs when two spacecraft come within a specified distance of each other. This distance can be defined in any way, but should include regions with a high collision probability density. The second definition is combined hardbody radius, r_{HB} . The combined hardbody is simply the combined volume of each of the spacecraft. The combined hardbody radius assumes that the volume of each spacecraft is spherical, and it is obtained by simply summing the hardbody radii of each of the spherical spacecraft.

Collision probability can be categorized into two main regimes, short-term and long-term. Short term, or linear cases as they are often referred to in the literature, are high velocity encounters that last a short period of time. Encounters with space debris usually fall into this category. For these encounters many simplifying assumptions may be made. First, the relative trajectories between spacecraft can be closely approximated by assuming rectilinear motion with a constant relative velocity. Second, the covariance changes very little over the course of the encounter. As such, very little error is incurred by assuming that it remains constant over

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the course of the encounter. Much work and many papers have been published regarding collision probability estimation for short term encounters. However, the assumptions made for the short-term case do not apply to clustered flight in which the amount of time the spacecraft spend in conjunction is significant, possibly lasting years. Techniques do exist to estimate collision probability for the long-term case. One of the most straightforward methods for estimating collision probability is to perform Monte Carlo analysis. Although Monte Carlo analysis is robust and simple to implement, it is not suitable for onboard calculations because it is computationally intensive. However, other methods have been developed which quickly estimate the probability of collision for long-term encounters^{9,10}. These methods avoid Monte Carlo by estimating the volume carved out by the combined hardbody through the probability density function, PDF, given by Eq. 1¹⁰.

$$P_c = \frac{1}{(2\pi)^{3/2} |P_{rr}|^{1/2}} \iiint_{vol} \exp \left[-\frac{1}{2} \mathbf{r}^T P_{rr}^{-1} \mathbf{r} \right] dx dy dz \quad (1)$$

where \mathbf{r} is the relative position vector and P_{rr} is the 3×3 position covariance matrix. Although these methods are quick, they lack the robustness that is required to accurately predict collision probability across a broad range of scenarios, and can at times be off by orders of magnitude⁹. For example these methods cannot accurately predict collision probability for the string-of-pearls configuration described later. Additionally, these methods do not properly estimate orbits that self intersect, or in other words trajectories that loop back upon themselves in the local relative frame. For comparative purposes, results will be shown for one of these methods¹⁰ referred to as Patera's method in this paper. Another approach is taken by Chan in which an analytic solution is obtained for the probability of collision³. His method is purely analytic and as such is very efficient, however is limited to a few special trajectories for which an analytic solution has been obtained. As a result, his method cannot be applied to the full range of relative trajectories encountered in cluster flight. Each of the methods discussed here and the methods that follow assume the state uncertainty is gaussian.

COLLISION PROBABILITY METRICS

In seeking to obtain a suitable metric for real-time on-board estimation of collision probability, two main considerations were taken into account. First, the algorithm must be quick enough so that it can be executed on flight hardware and in a limited amount of time. Second, the algorithm must be sufficiently accurate and robust so that the results are both meaningful and dependable. A variety of collision probability metrics were considered, three of which were considered suitable for on-board use. Fortunately, the three selected metrics share computations, thus computing all three requires little more effort than generating results for just one.

Minimum Mahalanobis Distance

The Mahalanobis distance, shown in Figure 1, is a measure of the number of standard deviations, σ_{LOS} , between two spacecraft along the line-of-sight vector⁶. Note that the distance between the module hardbodies is r' , and the distance between the module centers is r . The Mahalanobis distance is a simple metric that can be used to compute an upper bound, p_M , to collision probability. This upper bound is computed using the minimum value of the Mahalanobis distance, d_M , over the time period of interest. By projecting the probability density function onto the line of site vector, Figure 2, it can be seen how the minimum Mahalanobis distance can be used to compute an upper bound to collision probability. This upper bound corresponds to the area of the tails, defined by the minimum Mahalanobis distance, of the projected pdf. This metric requires little computation and can efficiently be implemented on-board a spacecraft. Once both the state and state uncertainty have been propagated forward for the time period of interest, the computation of the Mahalanobis distance requires little additional computational effort. The Mahalanobis distance and its associated upper bound to collision probability are given by Eqs. 2 and 3 respectively.

$$d_M = \left(1 - \frac{r_{HB}}{\|\mathbf{r}\|} \right) \sqrt{\mathbf{r}^T P_{rr}^{-1} \mathbf{r}} \quad (2)$$

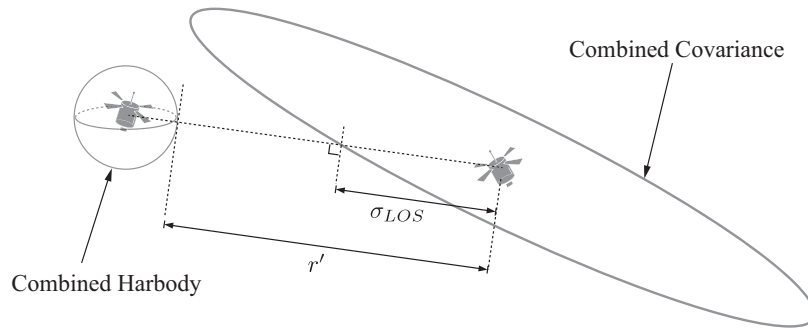


Figure 1. The metric based on Mahalanobis distance.

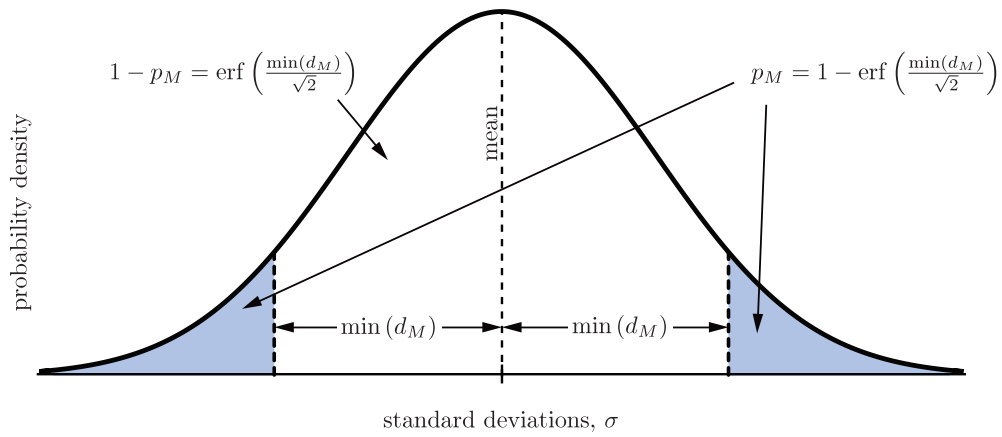


Figure 2. Probability upper bound given the minimum Mahalanobis distance.

where $\|r\|$ is the Euclidian norm of the relative position vector. Note that the Mahalanobis distance has been modified by shortening the relative position vector so that it does not include the hardbody radius.

$$p_M = \operatorname{erfc}\left(\frac{\min(d_M)}{\sqrt{2}}\right) \quad (3)$$

For many cases the probability of collision obtained using the Mahalanobis distance gives an upper bound that is far too conservative. For such cases additional metrics are need to compute the probability of collision. On the other hand, for lower probability cases, the Mahalanobis distance can quickly and definitively *prove* that the collision probability is extremely low.

Maximum Instantaneous Probability of Collision

The second metric selected to run on-board the spacecraft is the instantaneous probability of collision, p_I , given by Eq. 4.

$$p_I = \frac{\frac{4}{3}\pi r^3_{HB}}{\sqrt{(2\pi)^3 |P_{rr}|}} \exp\left(-\frac{1}{2}\mathbf{r}^T P_{rr}^{-1} \mathbf{r}\right) \quad (4)$$

This metric is the probability that a collision will occur at an instant in time. This metric is always lower than the total cumulative probability of collision and as a result it provides a lower bound on collision probability. The maximum instantaneous probability, $\max(p_I)$, provides a lower bound that is closest to the true probability of collision. This metric assumes that the local slope of the probability density function is constant. For practical applications this assumption results in little error. This metric generally agrees more closely to the true probability of collision obtained using Monte Carlo than the Mahalanobis distance does, however together with the Mahalanobis distance these two metrics provide a range in which the true probability of collision is guaranteed to fall. The instantaneous probability of collision and the Mahalanobis distance require common calculations, and as a result it requires little extra computation to compute them both.

Hybrid Probability

While the minimum Mahalanobis distance provides an upper bound to collision probability, and the maximum instantaneous probability of collision provides a lower bound to collision probability, it is desirable to have a metric to directly predict the probability of collision. A new metric was developed that is a combination of the previous two metrics. A least squares fit was performed on a diverse set of data shown in Figure 7. The result of the least squares fit or hybrid probability, p_H , is a weighted log average of the Mahalanobis distance and the maximum instantaneous shown in Eq. 5.

$$p_H = \exp(0.16 \ln(p_M) + 0.8 \ln(p_I)) \quad (5)$$

This final metric requires little additional computation to implement and provides an improved estimate for cases in which the Mahalanobis distance provides an upper bound to collision probability that is too conservative to be of value.

COMBINING PROBABILITIES

When computing collision probability, the standard method is to first compute the probability of collision between module pairs. This is a useful form because one can quickly identify which modules in the cluster are at the greatest risk of colliding. However one may wish to know the total risk that a collision will occur between any pair of modules in the cluster. Similarly, collision probability is first computed for a single control cycle. If the desired time frame for predicting collision probability involves multiple control cycles, one must combine the probability from each control cycle to obtain the total probability over the period of interest. Throughout the paper, a control cycle refers to the time span from one maneuver or navigation update to the next. Methods are presented for combining the probabilities for multiple module pairs and multiple control cycles.

Multiple Module Pairs

The probability of collision, p_i , is first computed for each possible module pair combination over a control cycle. Each of these probabilities must then be combined to obtain the probability, p_j , that at least one collision will occur within the cluster. Simply summing the probabilities for each of the possible combinations is incorrect and can lead to probabilities greater than one. The correct method is to calculate the probability that none of the modules will collide and subtract it from one

$$p_j = 1 - \prod_{i=1}^m (1 - p_i) \quad (6)$$

where m is the number of module pairs.

Multiple Control Cycles

The probability of a collision, p_j , within the cluster is computed over each control cycle. Each of these probabilities must then be combined to obtain the cumulative probability, p_{total} , that at least one collision will occur during at least one of the control cycles. Simply summing the probabilities for each of the control cycles is incorrect and can lead to probabilities greater than one. The correct method is to calculate the probability that a collision will not occur during any of the control cycles and subtract the result from one.

$$p_{\text{total}} = 1 - \prod_{i=1}^n (1 - p_j) \quad (7)$$

where n is the number of control cycles.

This method can also be used when the probability of collision, p_1 , is given for a specified length of time, T_1 , and it is desirable to extrapolate the probability of collision, p_2 , for a longer time period, T_2 .

$$p_2 = 1 - (1 - p_1)^{\frac{T_2}{T_1}} \quad (8)$$

This extrapolation is more accurate when the initial time period, T_1 , spans multiple orbits and control cycles.

STATE AND STATE UNCERTAINTY PROPAGATION

Both the state and state uncertainty must be propagated forward in time to compute the collision probability metrics. This is the most computationally intensive aspect of computing the collision probability. The quickest and simplest method for propagating the state and covariance matrix is to use the Clohessy-Wiltshire (CW) equations of relative motion⁷. The CW equations linearize the relative dynamics of two space objects. They do not take into account non-linear effects or perturbing forces. For a more accurate representation of the dynamics, J2 and drag effects should be taken into account.

To propagate the covariance using the CW equations the relative covariance is required. Often times (e.g. absolute navigation) the relative covariance is not directly available from navigation. The covariance of each respective spacecraft may be summed together to obtain the relative covariance. Doing this assumes that the covariances are uncorrelated. Although this assumption isn't always true, correlations between the covariances tend to decrease collision probability⁸. By neglecting these correlations the resulting estimates of collision probability are more conservative than they would have otherwise been. This effect is mathematically demonstrated by looking at Eq. 9 which shows how covariances should be combined.

$$P = P_1 + P_2 - 2(P_{12}) \quad (9)$$

where P_{12} is the correlation between P_1 and P_2 . Neglecting P_{12} results in P_{rel} growing. It should be noted that additional transformations may be required to get the covariance into the desired frame.

If perturbing forces are to be included in the propagation of the covariance matrix, then it may be useful to propagate the covariance in the inertial frame. If this is the case, instead of needing the relative covariance

the absolute covariance is needed. If the absolute covariance cannot be obtained directly from the navigation filter a simple approximation is to divide the relative covariance by two and apply it to each spacecraft. Once the covariances have been propagated they are then added back together to obtain the relative covariance. Again, additional transformations may be required to get the covariance into the desired frame.

ALGORITHM

A method is proposed which combines the concepts and elements from the previous sections into a single algorithm which attempts to achieve an acceptable level of speed, accuracy, and memory usage. This is achieved by implementing a staged approach to collision probability estimation. A brief description of the algorithm is presented here.

1. Both the filter state, \mathbf{X}_0 , and filter covariance, P_0 , are received from navigation.
2. Propagate the state of each module pair over the desired time period. For higher altitudes, smaller cluster sizes, and stable configurations, this propagation may be effectively performed using the CW equations.
3. Once the position between modules has been propagated, the relative position between each possible module pair is also computed.
4. Propagate the covariance over the desired time period.
5. At each time step both the Mahalanobis distance and the instantaneous probability of collision is computed using Eq. 1 and Eq. 4 respectively. The minimum Mahalanobis distance and the maximum instantaneous probability over the period of interest are calculated.
6. Compute the upper bound to the total probability of collision using the minimum Mahalanobis distance as shown in Eq. 3. Eq. 6 is used to compute the probability of collision for each module with respect to all of the other modules. If the probability of collision for each module is acceptable then no further calculations are needed and the algorithm can exit.
7. Compute the lower bound to the total probability of collision using the maximum instantaneous probability of collision for each module using Eq. 6. Compute the hybrid probability of collision using Eq. 5.
8. If it is desired, the total probability of collision over all module pairs may be computed using Eq. 6.

A factor influencing how far into the future the probability of collision should be predicted is the control cycle length. Although techniques such as linear covariance¹ may be used to incorporate the effects of control into the covariance prediction, this will add more complexity than is desirable in real time on board a spacecraft. To avoid the problem of taking burns into account, it is suggested that the collision prediction simply be performed to measure the passive safety of the cluster. The collision probability should be calculated just prior to and after a control cycle occurs. The probability of collision is computed after a maneuver plan has been generated, just prior to the execution of a maneuver, in order to ensure that the maneuver is safe. Once the maneuver has been completed and a navigation update is received, the probability of collision should once again be computed to ensure that the trajectories are passively safe. For efficiency, this probability estimate might only be predicted till the next control cycle to ensure that the modules are passively safe until another collision probability estimate is obtained.

RESULTS

All of the results presented here are for a circular reference orbit, and the results are present in the RIC coordinate frame. The results are shown for three cluster configurations each made up of four modules. The first configuration shown in Figure 3 is the string-of-pearls. The modules are evenly spaced in-track, and their positions are fixed in the RIC frame. The second configuration, shown in Figure 4, forms a circle when

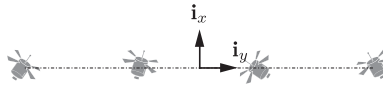


Figure 3. String-of-pearls cluster configuration as viewed from the radial in-track plane.

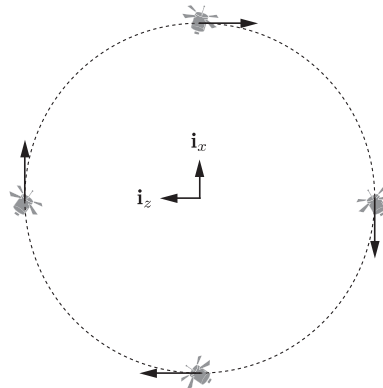


Figure 4. Circles in circle cluster configuration as viewed in the radial cross-track plane.

projected on the xz -plane. This configuration and the one that follows are passively safe because the modules will not collide by drifting in track^{4,5}. The final configuration, shown in Figure 5, is referred to in this paper as the cross configuration.

In order to test how far in the future the probability of collision should be predicted, six scenarios were ran. The hybrid probability of collision was estimated for varying lengths of time into the future. The results are shown in Figure 6.

The circles in circle configuration and the cross configuration are both passively stable, and near their steady state value within half an orbit for both the high and low drag cases. The string of pearls is not passively safe and does not approach its steady state value for a full orbit. How far in the future the probability of collision should be predicted is driven by the cluster configuration. If the cluster is designed to be passively safe, such that the projection of the modules onto the xz -plane do not overlap at any point along their nominal trajectories, then the length of time that the probability of collision should be calculated can be reduced. This is because the covariance primarily grows in-track. As a result the probability of collision for passively safe orbits grows very little after only a couple of orbits. For many cases the collision probability should be predicted for only a single orbit in advance. For cases where the cluster was not designed to be passively safe, and depending on the control strategy, it may be desirable to predict the probability for more than two orbits in the future.

Each of the collision probability metrics were computed for over 300 cases and compared to Monte Carlo. These cases include string-of-pearls, football orbit², and overhead flyby² and each were propagated using the CW equations. The results are shown in Figure 7. As can be seen by the results, the Mahalanobis distance does provide a conservative upper bound to collision probability. This metric at time, particularly when the true probability was high, is too conservative to be of value. It can be seen that the maximum instantaneous probability of collision does in fact provide a lower bound to collision which is within an order of magnitude for the results shown. Results for Patera's method are also shown. It can be seen that for most cases his

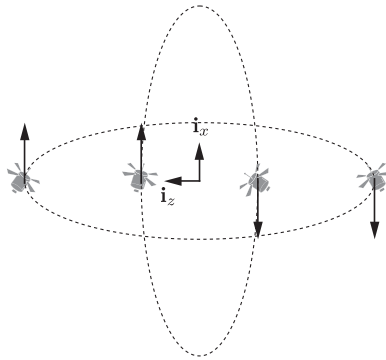


Figure 5. Cross cluster configuration as viewed in the radial cross-track plane.

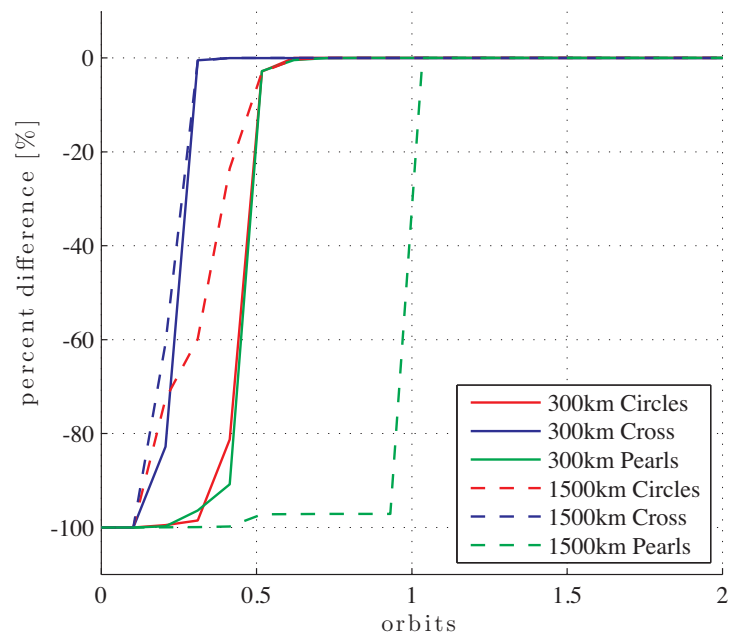


Figure 6. The percent difference between the cumulative probability for two orbits and the cumulative probability at each time step up until two orbits is shown. The probability is calculated for six trajectories using the hybrid probability.

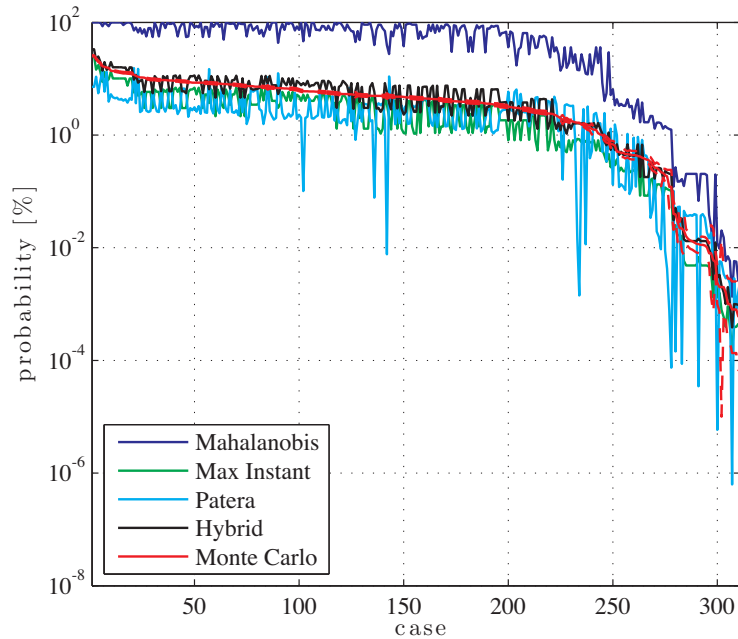


Figure 7. Result of various collision probability metrics.

method agreed well to Monte Carlo, but in some instances substantially underestimated the probability. The hybrid method offers a good blend of speed, accuracy, and versatility. It is the one method considered that is implementable across a diverse range of scenarios and that achieves the accuracy desired without requiring unrealistic computational resources. For every case ran, the hybrid method agrees with Monte Carlo to within an order of magnitude, and typically it agrees within a factor of two.

For clusters at higher altitudes the effects of perturbing forces are less prominent and the CW equations provide an adequate dynamic model. For clusters at lower altitudes significant error can be incurred by neglecting these effects. To obtain the best results at lower altitudes it is suggested that both J_2 and drag be considered. As can be seen from Table 1 the effect of these forces on probability of collision is more pronounced in the propagation of the state than in the propagation of the covariance. Case 1 of the results, where both the state and covariance are propagated using a dynamic model which includes J_2 and drag, is the closest to the true value, and is what the other results should be compared against. As can be seen by comparing cases 1 and 4, an estimate that is within an order of magnitude of the desired value is obtained by propagating the covariance using a point mass model and propagating the state using a higher fidelity model, which includes drag and J_2 . Cases 5 through 16, in which the covariance was not propagated using both J_2 and drag, provide worse estimates than cases 2 through 4.

CONCLUSIONS

The algorithm outlined in this paper can be used on-orbit to estimate collision probability for a cluster of satellites. The hybrid probability of collision metric is both efficient and accurate. It was shown that collision probability can be sufficiently estimated by limiting the integration time to one orbit in the future and by using a simplified dynamic model. Techniques were shown for combining probabilities for multiple module pairs and for multiple control cycles.

Table 1. Passive collision probability when propagating the state and covariance using different dynamic models. Each case is for a cluster size of 4 and a reference trajectory of 300 km.

Case Number	State		Covariance		Probability		
	J_2	Drag	J_2	Drag	Hybrid	Instantaneous	Mahalanobis
1	Yes	Yes	Yes	Yes	1.14E-07	1.19E-08	1.67E-04
2	Yes	Yes	No	Yes	2.96E-07	3.18E-08	4.87E-04
3	Yes	Yes	Yes	No	3.71E-07	3.96E-08	6.58E-04
4	Yes	Yes	No	No	6.62E-07	7.18E-08	1.26E-03
5	No	Yes	Yes	Yes	4.14E-10	3.58E-11	3.88E-07
6	No	Yes	No	Yes	1.94E-09	1.77E-10	2.09E-06
7	No	Yes	Yes	No	4.11E-09	3.79E-10	4.93E-06
8	No	Yes	No	No	1.01E-08	9.57E-10	1.31E-05
9	Yes	No	Yes	Yes	4.14E-12	3.76E-13	9.63E-10
10	Yes	No	No	Yes	3.88E-12	3.51E-13	9.00E-10
11	Yes	No	Yes	No	4.39E-12	3.99E-13	1.03E-09
12	Yes	No	No	No	4.10E-12	3.71E-13	9.60E-10
13	No	No	Yes	Yes	6.62E-07	7.18E-08	1.26E-03
14	No	No	No	Yes	4.44E-12	4.03E-13	1.04E-09
15	No	No	Yes	No	5.02E-12	4.58E-13	1.19E-09
16	No	No	No	No	4.69E-12	4.27E-13	1.11E-09

NOTATION

m	number of module pairs
n	number of control cycles
P_c	total probability of collision
p_H	hybrid probability of collision
p_I	instantaneous probability of collision
p_i	probability for single module pair
p_j	probability for multiple module pairs
p_M	upper bound to collision probability associated with the Mahalanobis distance
P_{rr}	relative position covariance
p_{total}	total probability over multiple control cycles
\mathbf{r}	relative position vector
r_{HB}	combined hardbody radius
σ_{LOS}	standard deviation along the line-of-sight vector

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